# Porous Versions of Kenderov's Continuity Phenomenon and a Topological Variational Principle

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**Abstract**

Kenderov (1983) proved that an arbitrary set-valued mapping from a topological space $X$ to a topological space $Y$ with countable base has some properties resembling continuity at every point of a residual subset $X_0 \subset$X (i.e. its complement $X \setminus X_0$ is of first Baire category). He proved a more restrictive version when $Y$ is a complete metric space. His statement has far-reaching consequences and can be called a "continuity phenomenon".

We show that, in the case when $X$ is a normed space, the set $X \setminus X_0$ is even $\sigma$- porous. It implies that several "generic" results have "$\sigma$- porous" versions, with unified proofs. For example, every *monotone* (and even *submonotone*) mapping on a separable Banach space is single-valued on the complement of $\sigma$- porous set, and every aproximate convex continuous function is Gateaux differentiable on the complement of $\sigma$- porous set. These results are valid also in a Banach space with a Gateaux differentiable norm. In Asplund spaces, analogous results are also strengthened to porous versions.

As other applications we present a weak form of Stegall's variational principle in the duals of separable Banach spaces and a porosity result for well posed optimization problems with continuous perturbations.

We generalize an abstract variational principle in Banach spaces, introduced by Topalova & Zlateva, by showing that the set $\mathbb{P}_0$ of perturbations for which a perturbed lower semi-continuous function $f$ is WPMC (Well Posed Modulus Compact) not only contains a dense $G_\delta$ subset, but is also a complement to a $\sigma$-porous subset in a specifically defined perturbation space $\mathbb{P}$. Moreover, if the space is a Musielak-Orlicz sequence space satisfying $\ell_\Phi \cong h_\Phi$, then the notion WPMC is replaced by the stronger notion of Tikhonov well posedness, if the domain of the function under consideration is in the positive octant.

We give several applications. The first one is that the Musielak-Orlicz sequence spaces have the Radon-Nikodym property and, therefore, are dentable by proving the validity of Stegall's variational principle, implying that their duals are $w^*$-Asplund. We establish also a sufficient condition such spaces to be Asplund. Next we determine the type of the smoothness of such spaces under suitable conditions.

One of the key instrument for proving these results is the porous version of the continuity phenomenon.

We present a new variational principle in completely regular topological spaces with dense completely metrizable subspaces. The proposed variant is similar to

the Ekeland variational principle, but instead of the norm, a special function constructed from Urisson functions is used. As applications, known generic result for Tikhonov well posedness of optimization problems are strengthened: dense $G_\delta$ sets are replaced by complement of sigma-porous sets in a space of continuous functions.